

Simultaneous and Sequential Synchronisation in Arrays

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Abstract. We discuss the possibility of simultaneous and sequential synchronisation in vertical and horizontal arrays of unidirectionally coupled discrete systems. This is realized for the specific case of two dimensional Gumowski-Mira maps. The synchronised state can be periodic, thereby bringing in control of chaos, or chaotic for carefully chosen parameters of the participating units. The synchronised chaotic state is further characterised using variation of the time of synchronisation with coupling coefficient, size of the array etc. In the case of the horizontal array, the total time of synchronisation can be controlled by increasing the coupling coefficient step wise in small bunch of units.

PACS. 05.45.Xt Synchronization; coupled oscillators – 05.45.-a Nonlinear dynamics and nonlinear dynamical systems

1 Introduction

Synchronisation of the dynamical variables of coupled systems is an important nonlinear phenomenon where intense research is being concentrated recently [1]. This is probably because of its engineering applications like spread spectrum and secure data transmission using chaotic signals [1,2,3], control of microwave electronic devices [4], graph colouring etc. Also communication between different regions of the brain depends heavily on the synchro-

nised behaviour of neuronal networks [5,6]. Moreover patterns of synchrony and phase shifts in animal locomotion is gaining importance as a field of active study [7,8,9,10,11,12]. In general, the synchronised networks for analysing or modelling all these physical or biological situations are constructed by coupling basic dynamic units with a well defined connection topology that can be nearest neighbour, small world, random or hierarchical architectures. In addition, in specific applications like communication or neuronal networks, a realistic modelling may require the introduction of connection delays due to finite information

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transmission or processing speeds among the units [13]. In any case, it is found that the collective dynamics depends crucially on the connection topology [14].

The simplest yet the most widely used topology in this context is the linear array and its combinations. The study of synchronisation in arrays of systems was first applied to laser systems [15,16] which has relevance in optical communication systems. Since then the occurrence of synchronisation in coupled map lattices has been extensively studied with many consequent applications [17]. Such systems, with synchronisation in temporally chaotic but spatially ordered units forming an array, is applied in many situations like data driven parallel computing [18]. However most of these cases studied so far involve continuous systems of chaotic oscillators.

In this paper, we consider two such regular arrays, one vertical and the other horizontal, that works under the drive response mechanism, where the connection is unidirectional. We find that the former setup leads to simultaneous synchronisation while the latter results in sequential synchronisation. Here we would like to comment that in most of the connected networks, the synchronisation is found to occur simultaneously. However the topology in the linear horizontal array introduced here develops synchronisation sequentially and the delay time from one unit to the next can be adjusted by external control. This mechanism therefore would be useful for many technological applications. These two types of synchronisations are characterised using response time (which is the time for synchronisations to stabilise), size effect, bunching effect

etc. These two arrays can be further worked together to produce square lattice networks with desirable or useful inter connections.

The array is realised here with a two dimensional discrete system or map as the local unit and a connection that involves a non linear function forming part of the map function. The stability of the simultaneously synchronised state for the vertical array is studied by computing the Maximum Conditional Lyapunov Exponent (MCLE) [19], so that the minimum coupling coefficient required for onset of synchronisation can be deduced. The dependence of the characteristic response time τ_s on the coupling coefficient ϵ is analysed numerically. A horizontal array with the same dynamics is constructed with each unit driven by the previous one, modelling an open flow system and leading to sequential synchronisation. In this case the time taken for the last unit to synchronise is taken as the total response time τ_s . The behaviour of its average for different initial conditions and size N of the system are studied. The additional time or delay time τ_l required for the last unit to synchronise after its previous one has synchronised is found to saturate with system size. Moreover we note an interesting bunching effect where the total τ_s can be controlled by varying the value of ϵ in bunch of m units.

In Section 2, we introduce the basic unit which serves as the driving as well as the driven systems with identical individual dynamics. The concept of generalised synchronisation and its stability in the context of unidirectionally coupled systems is also discussed. The construction and the collective dynamics of the vertical array and the

characterisation of simultaneous synchronisation is given in section 3. In section 4, we introduce sequential synchronisation and its control due to the bunching effect of the unidirectionally coupled units. Our concluding remarks are given in section 5.

2 Basic Dynamical unit and Generalised Synchronisation

The basic unit used here for the present analysis of synchronisation in arrays is a two dimensional discrete systems, which serves both as the driving and driven systems defined in the phase space $\overline{X}(n) = (X(n), Y(n))$. The specific system chosen for this work is the Gumowski-Mira recurrence relation [20] given as

$$\begin{aligned} X(n+1) &= Y(n) + a \left(1 - bY(n)^2\right) Y(n) + f(X(n)), \\ Y(n+1) &= -X(n) + f(X(n+1)). \end{aligned} \quad (1)$$

where $f(X(n)) = \mu X(n) + \frac{2(1-\mu)X^2(n)}{1+X^2(n)}$ and n refers to the discrete time index.

Our earlier investigations in this system reveal that (1) is capable of giving rise to many interesting two dimensional patterns in (X, Y) plane that depend very sensitively on the control parameter μ [21]. This can be exploited in decision making algorithms and control techniques for computing and communications. We have tried three different coupling schemes in two such systems [22] and found that they are capable of total or lag synchronisation in periodic, quasi periodic or chaotic states, when N such systems are geometrically set to form a vertical

or horizontal array and driven unidirectionally, they are capable of synchronising to the same chaotic state.

In the context of unidirectionally coupled systems, the type of synchronised behaviour called generalised synchronisation has been attracting much attention recently [23, 24]. Here the states of the driving system \overline{X}_d and the driven system \overline{X}_{dr} are dynamically related by a function F such that the relation $\overline{X}_{dr}(t) = F(\overline{X}_d(t))$ is true once the transients are over. The form of F can be smooth or fractal and in either case, the procedure for finding the same can be complicated. Hence often an auxiliary system identical to the driven system is introduced as $X_a(t)$. The initial conditions of X_{dr} and X_a are taken different (both being individually chaotic in dynamics) but lying in the basin of the same attractor. Once the transients have settled, the dynamical equivalence of $X_{dr}(t)$ and $X_a(t)$ is taken as an indication of generalised synchronisation between $X_d(t)$ and $X_{dr}(t)$.

3 Simultaneous Synchronisation in a Vertical Array

We extend the above concept to construct a vertical array of N identical systems, $[X_{dr}^1(n), X_{dr}^2(n) \cdots X_{dr}^N(n)]$; each driven independently by $X_d(n)$. All the systems are identical and individually evolve according to (1). Fig. 1 show the above scheme of construction of vertical arrays.

Here the driving system follows the dynamics

$$\begin{aligned} X_d(n+1) &= Y_d(n) + a \left(1 - bY_d^2(n)\right) Y_d(n) + f(X_d(n)), \\ Y_d(n+1) &= -X_d(n) + f(X_d(n+1)). \end{aligned} \quad (2)$$

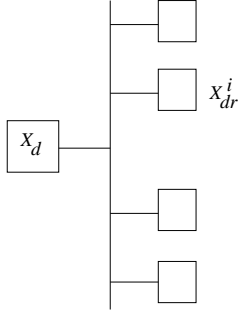


Fig. 1. Schematic view of the construction of a vertical array

$$\text{with } f(X_d(n)) = \mu_d X_d(n) + \frac{2(1-\mu)X_d^2(n)}{1+X_d^2(n)}.$$

The i^{th} driven unit in the vertical array has the dynamics

$$\begin{aligned} X_{dr}^i(n+1) &= Y_{dr}^i(n) + a \left(1 - bY_{dr}^{i2}(n)\right) Y_{dr}^i(n) \\ &\quad + f(X_{dr}^i(n)) + \epsilon \left(f(X_d(n)) - f(X_{dr}^i(n))\right) \\ Y_{dr}^i(n+1) &= -X_{dr}^i(n) + f(X_{dr}^i(n+1)). \end{aligned} \quad (3)$$

with $f(X_{dr}^i(n)) = \mu_{dr} X_{dr}^i(n) + \frac{2(1-\mu_{dr})X_{dr}^{i2}(n)}{1+X_{dr}^{i2}(n)}$ where ϵ is the coupling coefficient of the unidirectional coupling applied to the X variable through the function $f(x)$. The parameters a and b are set as $a = 0.008$ and $b = 0.05$. The total number of units considered is $N = 51$. The value of μ_{dr} is chosen to be the same for all the 50 driven units. We can realise synchronisation for different combinations of values of μ_{dr} and μ_d with μ_d , in general different from μ_{dr} . For the special case of $\mu_d = \mu_{dr}$ all the 51 units synchronise including the driving system, when started with different initial conditions.

Fixing the value of coupling coefficient $\epsilon = 0.9$, the values of μ_d, μ_{dr} for which synchronisation is feasible in the 50 driven systems is plotted in fig. 2. In the parameter plane considered here in the range $-0.2 < \mu_d < -0.5$, $-0.2 < \mu_{dr} < -0.5$, the points marked $*$ indicates (μ_d, μ_{dr}) values leading to synchronised periodic state with

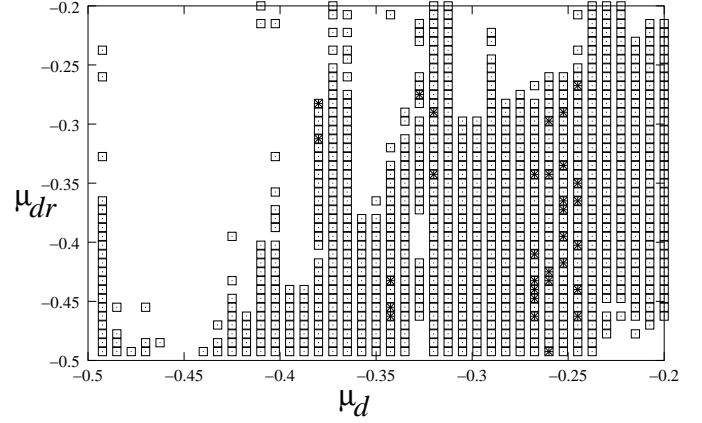
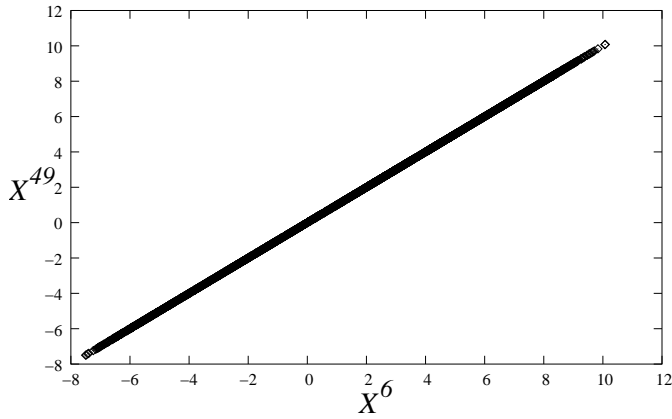


Fig. 2. Points in the $\mu_d - \mu_{dr}$ plane for which synchronisation of the driven systems are possible with $\epsilon = 0.9$. Points marked $*$ indicates μ_d, μ_{dr} values leading to synchronised states with periodicity less than 15. Points marked \square correspond to synchronisation in higher periodic states or mostly chaotic states.

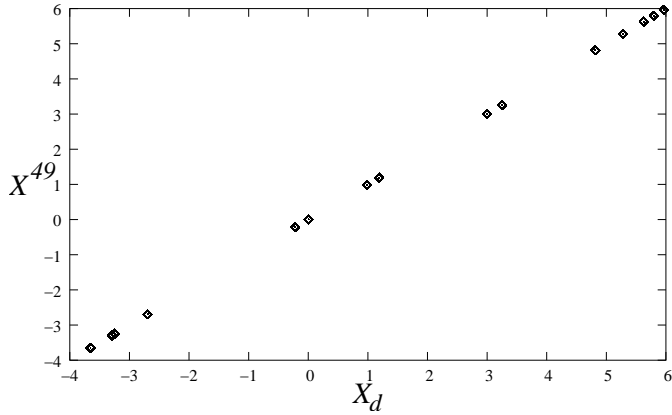
periodicity less than 15. Points marked \square indicates synchronisation in higher periodic state or mostly chaotic states.

For specific cases like $\mu_d = \mu_{dr} = -0.39$ both the driving system and driven systems are in chaotic state individually. With $\epsilon = 1.56$ all the 50 driven systems are synchronised in the chaotic state while the driving system is asynchronous with them. However when ϵ is slightly increased to 1.6 all the 51 units are found to synchronise in the chaotic state. Fig. 3a gives this chaotic synchronisation between two participating driven systems for $\epsilon = 1.6$, where the iterates of the X variable of the 6th and 49th units are plotted, after the transients have died out.

For $\mu_d = \mu_{dr} = -0.23$, individually the systems are chaotic. For $\epsilon = 0.9$, all the N driven systems are synchronised to the same periodic state of periodicity 15. But the driving system is also synchronised only when ϵ is in-



(a)



(b)

Fig. 3. (a) Chaotic synchronisation between two participating driven systems with $\epsilon = 1.6$, $\mu_d = \mu_{dr} = -0.39$. Here the iterates of the X variable of the 6th and 49th units are plotted. (b) Synchronised periodic 15 cycle for $\mu_d = \mu_{dr} = -0.23$ with $\epsilon = 1.6$. The iterates of the X variable of the driving system and 49th driven unit is plotted.

creased to 1.6. Fig. 3b gives this synchronised periodic 15 cycle for $\epsilon = 1.6$, where the iterates of the driving system and the 49th unit are plotted. Thus the driving system and the driven system can be simultaneously synchronised only when $\mu_d = \mu_{dr}$ and when ϵ is very large ie $\epsilon \sim 1.6$.

For $\mu_d = -0.2$, the driving system is in periodic 8 cycle. For $\mu_{dr} = -0.39$ the driven systems are individually chaotic. For $\epsilon = 0.9$ all the N driven systems are synchronised in the chaotic state. Fig. 4 shows the synchronised chaotic state for the above case. Here the iterates of the X variable of the 48th unit and 10th unit are plotted after the transients have died out.

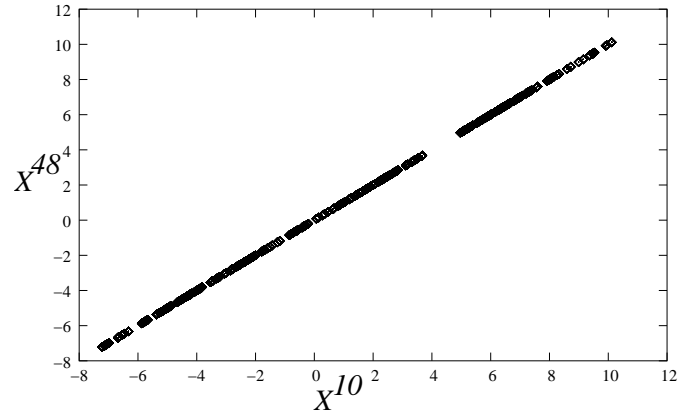


Fig. 4. Synchronised chaotic states between two driven systems for $\mu_d = -0.2$, $\mu_{dr} = -0.39$ with $\epsilon = 0.9$. Here the iterates of the X variable of the 48th unit and the 10th units are plotted.

The condition for the stability of generalised synchronisation is discussed using the Maximal Conditional Lyapunov Exponent λ_{MCLE} [25,26]. Here the Lyapunov Exponent of the driven system is calculated and it is different from the uncoupled system, since it depends on the dynamics of the driving system also. The condition for the stability of generalised synchronisation is that λ_{MCLE} should be negative [27]. From equations (2) and (3) the

Jacobian matrix for the i^{th} unit can be written as

$$M = \begin{bmatrix} (1 - \epsilon) \frac{\partial F^i}{\partial X^i} & \frac{\partial F^i}{\partial Y^i} \\ \frac{\partial G^i}{\partial X^i} & \frac{\partial G^i}{\partial Y^i} \end{bmatrix} \quad (4)$$

where $X_{dr}^i(n+1) = F^i(X, Y)$ and $Y_{dr}^i(n+1) = G^i(X, Y)$.

If σ^1 and σ^2 are the eigen values of the product of the Jacobian matrices at every iteration such that $\sigma^1 > \sigma^2$, then [28]

$$\lambda_{MCLE} = \lim_{m \rightarrow \infty} \frac{1}{m} \ln |\sigma^1| \quad (5)$$

λ_{MCLE} can be calculated numerically for different ϵ values using (5).

We consider the case $\mu_d = -0.2$ and $\mu_{dr} = -0.39$ which we have discussed above and calculate λ_{MCLE} for different ϵ values. Calculations are done for 10000 iterates after leaving initial 70000 iterates as transients. In fig. 5 the values of λ_{MCLE} for different ϵ values are plotted. It

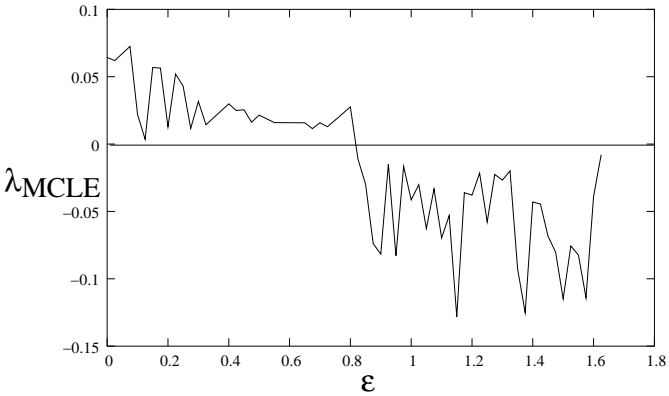


Fig. 5. The variation of Maximum Conditional Lyapunov Exponent (λ_{MCLE}) of the driven system with the coupling coefficient ϵ . Here $\mu_d = -0.2$, $\mu_{dr} = -0.39$. The minimum coupling coefficient for synchronisation $\epsilon_{\min} = 0.829$.

is found that, λ_{MCLE} crosses zero at $\epsilon = 0.829$, which is the minimum value of ϵ [19] viz ϵ_{\min} such that for $\epsilon > \epsilon_{\min}$ the synchronised state is stable.

For the above case the coupling coefficient is varied in steps from 0.84 to 0.97 and the time taken for reaching synchronisation in the driven systems is noted. Fig. 6 gives the variation of thus average response time τ_s (averaged

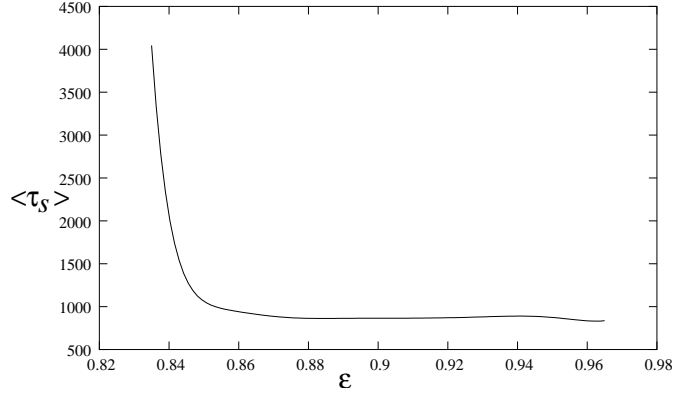


Fig. 6. The variation of the response time $\langle \tau_s \rangle$ which is the total time for synchronisation with coupling coefficient ϵ for the 50th unit. $\langle \tau_s \rangle$ is almost constant for values of $\epsilon > 0.87$.

over 10 different initial conditions) with ϵ for the 50th unit. It is interesting to note that $\langle \tau_s \rangle$ is almost constant for values of $\epsilon > 0.87$. In this case since the synchronisation is simultaneous and coupling is unidirectional and similar, the average $\langle \tau_s \rangle$ is independent of the size of the array N .

4 Sequential synchronisation in a Horizontal Array

In this section a horizontal array of N identical systems with open ends, where each unit is driven by the previous one is introduced. The coupling is through the nonlinear function $f(X, Y)$ as in the previous case. Fig. 7 gives the schematic view of unidirectional coupling in a flow which consists of N units.

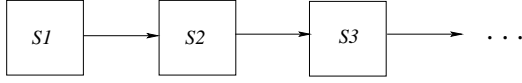


Fig. 7. Schematic view of unidirectional coupling in a flow system.

The i^{th} unit in the horizontal array follows the dynamics

$$\begin{aligned}
 X^i(n+1) &= Y^i(n) + a \left(1 - bY^{i^2}(n) \right) Y^i(n) \\
 &\quad + f(X^i(n)) + \epsilon \left(f(X^{i-1}(n)) - f(X^i(n)) \right) \\
 Y^i(n+1) &= -X^i(n) + f(X^i(n+1)).
 \end{aligned} \tag{6}$$

with $f(X^i(n)) = \mu X^i(n) + \frac{2(1-\mu)X^{i^2}(n)}{1+X^{i^2}(n)}$.

The control parameter μ is the same for all the units such that the units are chaotic individually. This set up is found to give rise to sequential synchronisation in the array. In our calculations we consider an array of 51 units. This can be extended to any number of units N .

For $\mu = -0.23$ where the individual systems are chaotic, and coupling coefficient $\epsilon = 1.9$, we find that synchronisation sets in sequentially with the 2nd synchronising after the first, the third after the second and so on. The time taken by the last unit to synchronise is taken as $\langle \tau_s \rangle$ which is the average total response time for the whole array. Fig. 8 shows this synchronised chaotic state after the last unit has synchronised. The $\langle \tau_s \rangle$ is found to vary with coupling coefficient ϵ as shown in Fig. 9. It is found that $\langle \tau_s \rangle$ has a minimum value for a particular ϵ which in this case is $\epsilon = 2$.

The delay time τ_l ie, the additional taken for the N^{th} unit to synchronise after its previous one has synchronised is defined as $\tau_l = \tau_s^N - \tau_s^{N-1}$. This τ_l is found to saturate

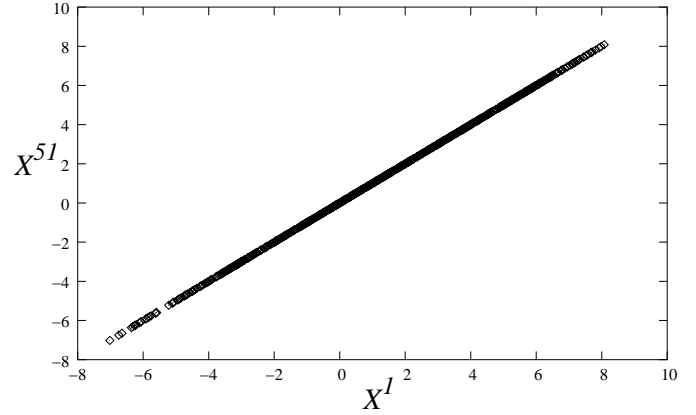


Fig. 8. Synchronised chaotic state for $\mu = -0.23$ with $\epsilon = 1.9$.

The iterates of the X variable of the first unit and the last unit (51st unit) are plotted.

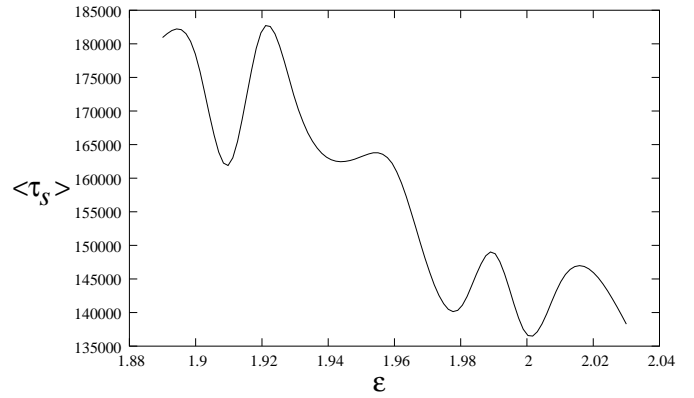


Fig. 9. The variation of total response time $\langle \tau_s \rangle$ of the 51st unit with coupling coefficient ϵ . $\langle \tau_s \rangle$ has a minimum value for $\epsilon = 2$.

with the system size [29] as shown in fig. 10. Beyond $N = 35$, τ_l is almost constant.

An interesting observation in this horizontal array of units is a bunching effect that reflects in the total response time $\langle \tau_s \rangle$. For this instead of fixing the same value for the coupling coefficient ϵ for all the units, we fix its value for a particular number of units and increase it in steps for the next bunch and so on. Then the total $\langle \tau_s \rangle$ is found to be smaller compared to the previous case of the same ϵ for

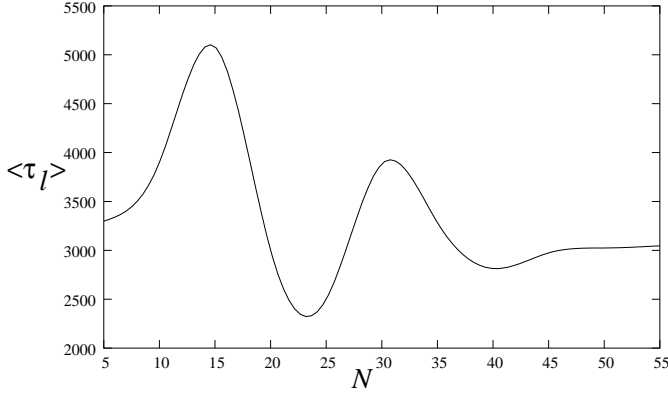


Fig. 10. The delay time τ_l which is the additional time taken for the N^{th} unit to synchronise after its previous one has synchronised is found to saturate with system size N . Beyond $N = 35$, τ_l is almost a constant.

all the units. Moreover this time depends on the size of the bunch and is minimum for a certain number of units in each bunch.

We report a few specific cases. With $\mu = -0.23$ the value of ϵ is increased in steps of 0.001 for each bunch so that ϵ for the last bunch is $\epsilon_{\text{max}} = 2.01$ for different bunch sizes. In each case the total response time $\langle \tau_s \rangle$ is found. Fig. 11 shows how the response time $\langle \tau_s \rangle$ changes with the variation in the bunch size m , i.e., number of units in each bunch. The response time $\langle \tau_s \rangle$ is minimum when the bunch size $m = 8$. For $m = 8$, $\langle \tau_s \rangle = 138612$ iterations, whereas when $\epsilon = \epsilon_{\text{max}} = 2.01$ for all the units, the response time $\langle \tau_s \rangle = 144404$ iterations.

As a second case for same $\mu = -0.23$, ϵ_{max} is taken as 1.91 and calculations repeated as above. In this case $\langle \tau_s \rangle$ is found to be minimum and is 152150 iterations when the size of the bunch is $m = 7$ as shown in Fig. 11. If $\epsilon = 1.91$ for all the units, $\langle \tau_s \rangle$ is 161953 iterations. We observe that the decrease in $\langle \tau_s \rangle$ for the whole array due

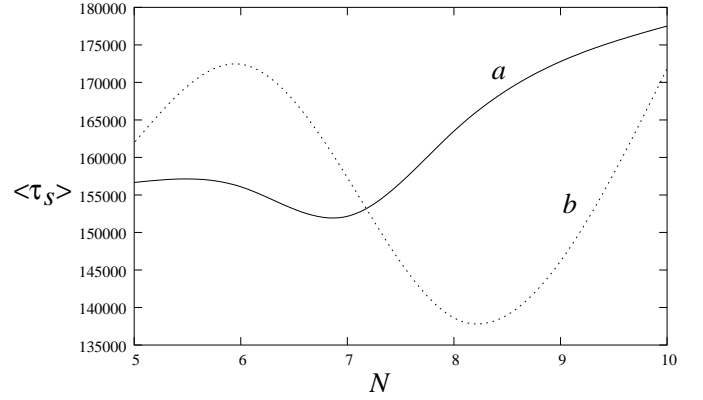


Fig. 11. Change in the response time $\langle \tau_s \rangle$ of the last unit with bunch size m . Here $\mu = -0.23$. Curve *a* is for $\epsilon_{\text{max}} = 1.91$, when the bunch size $m = 7$, the response time $\langle \tau_s \rangle$ is minimum. *b* is for $\epsilon_{\text{max}} = 2.01$, the response time $\langle \tau_s \rangle$ is minimum for $m = 8$. In both cases $\langle \tau_s \rangle$ is found to be less than the case when we apply ϵ_{max} to all the units.

to bunching must be reflected in the response time of each bunch. So for the minimum case, the response times for the last unit of the first bunch (i.e., 8th unit), last unit of the second bunch (i.e., 16th unit) etc. are noted with bunching. The same quantity with ϵ same for all the units i.e., without bunching are also noted. $\langle \tau_s \rangle$ thus obtained are plotted against the respective units in Fig. 12. It is found that except for the 8th unit in the first bunch the response time is less in the case of bunching.

5 Conclusion

In this work we report how synchronisation in an array of systems can be made more efficient and flexible to suit specific applications. We consider two such arrays, vertical and horizontal, working under the drive-response mecha-

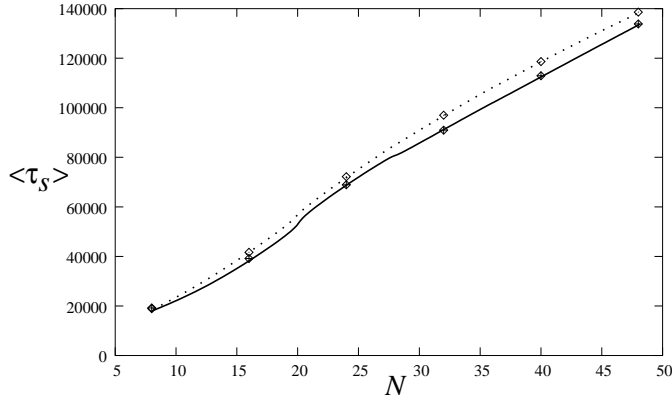


Fig. 12. The response time $\langle \tau_s \rangle$ is plotted for different units for $\mu = -0.23$, $\epsilon_{\max} = 2.01$. The dotted line gives the total response time $\langle \tau_s \rangle$ of 8th unit, 16th unit, 24th unit etc. when $\epsilon_{\max} = 2.01$ is applied to all the units. Full line gives the total response time $\langle \tau_s \rangle$ of 8th unit (last unit of 1st bunch), 16th unit (last unit of 2nd bunch), 24th unit (last unit of 3rd bunch) etc. when ϵ is increased in steps for each bunch so that $\epsilon_{\max} = 2.01$ for the last bunch. Here $2.005 \leq \epsilon \leq 2.01$ with step size 0.001. Thus bunching can control the total response time $\langle \tau_s \rangle$ of the array.

nism with a two-dimensional discrete systems as the unit dynamics.

It is observed that synchronisation sets in all the systems simultaneously in the vertical setup. The minimum value of the coupling coefficient ϵ required for stability of synchronisation is computed numerically from the Maximum Conditional Lyapunov Exponent. The possible choices of parameters for stable states of synchrony are isolated. The specific cases of chaotic single units synchronising to periodic and chaotic synchronised states and periodic single units stabilising to chaotic states of synchronisation are considered in detail. The average response time required

to overcome the transients is found to saturate beyond a certain value of ϵ . The horizontal array exhibits many interesting features useful for technological applications. In this case the synchronisation sets in sequentially from unit to unit along the array since the coupling is unidirectional. The total response time for the whole array has a minimum as ϵ is varied. The additional time required for the last unit to synchronise after the previous one is found to saturate with system size.

We further note that the total response time for the whole array can be reduced by introducing bunching with step wise increase of ϵ from bunch to bunch. There exists a specific bunch size giving minimum time which depends on the choice of the parameter and the maximum ϵ given to the last bunch. This makes the sequential synchronisation flexible and controllable to suit specific applications.

At present we do not find any specific reasons for the above findings to depend on the unit chosen. For different choices of the unit dynamics the behaviour should be qualitatively similar. To establish this generality and applicability of the present technique, we are trying it out for a number of systems. The results will be published elsewhere.

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